# The Determination of Optimum Flight Profiles for Short-Haul Routes

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**A method of identifying suboptimum flight profiles for short-haul propeller-driven aircraft, using multivariate optimization techniques, is outlined. These profiles are defined in terms of simple parameters which can be readily monitored and controlled by the crew without the assistance of an autopilot. A sequence of minimum-fuel studies, based on a typical modern commuter aircraft, is presented. The effects of some operational constraints are examined. It is found that a 1.5% fuel saving can be achieved relative to the current flight-manual operating techniques. The climb and descent segments are most significant in this type of operation.**

## Nomenclature

$C_j$ = $j$th constraint
$C_P$ = power coefficient
$C_T$ = thrust coefficient
$D_{\text{prop}}$ = propeller diameter, m
$H$ = cruising height, m
$J$ = advance ratio
$N$ = propeller rpm
$S$ = distance, km
$T$ = throttle setting
$V$ = indicated airspeed, m/s
$x_i$ = $i$th variable
$\boldsymbol{x}^{(k)}$ = vector of variables at $k$th search step
$\rho$ = air density, kg/m$^3$

*Subscripts*

$A,B,C$ = climb, cruise, and descent conditions, respectively

## Introduction

THE problem of generating a flight profile that will optimize a given objective function has been treated comprehensively by many investigators. The calculus of variations has been the primary tool for analyzing performance and developing appropriate control laws for optimal trajectories. Shultz and Zagalsky[1] presented an overview of such work using different levels of approximation to the aircraft motion, ranging from the full flight-path equations to the energy approximation. These methods are eminently suitable for treating marginally subsonic or supersonic jet aircraft performance, although it is necessary to use simplifications when dealing with the entire flight profile from initial to final point.

In all cases, it is assumed that the control variables will be time-dependent functions and an autopilot system would be available for following the optimal altitude-speed schedule.

When analyzing the performance of short-haul propeller-driven commuter aircraft the suitability of the above techniques may be questioned. This type of aircraft is rarely equipped with sophisticated autopilot systems, but, rather, is flown in accordance with common piloting techniques and handbook procedures. The pilot would be expected to maintain constant (but different) values of indicated airspeed (IAS) and throttle settings during each phase (climb, cruise, and descent) of the flight. At most, a switch in IAS/throttle might be specified, e.g., at a certain altitude during the climb. Cockpit workload and human tracking limitations would prevent a continuous variation of these variables with time.

Thus, the flight profile may be described in terms of a number of discrete-valued control variables. This suggests that a flight profile analysis algorithm can be coupled with a multivariate optimization (MVO) routine to determine the optimum vector of control variables. Apart from its applicability to operational techniques, such an approach offers the following additional advantages:

1) The entire flight may be analyzed as a complete problem, taking into account the interactions between the climb, cruise, and descent phases. This is particularly significant in short-haul operations where classical cruise optimization techniques are inappropriate. Indeed a cruise phase may not exist if the stage is short.

2) The mathematical modeling of the thrust characteristics of propeller-driven aircraft presents no problems. Past approaches have been restricted to the simplified thrust models associated with jet aircraft to ease the mathematical analysis. The possibility of utilizing propeller rpm as a further control variable also presents itself.

3) Operational limitations can be incorporated more easily into the model by utilizing the ability of the MVO routine to handle all types of constraints. Such constraints can represent either physical limitations of the aircraft model or air-traffic control (ATC) and environmental limitations that the user may wish to impose.

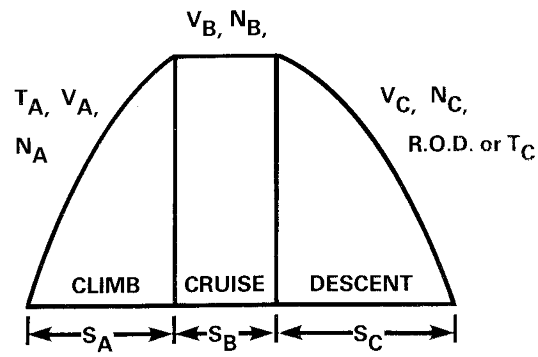


Fig. 1 Typical profile and control variables.

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Temperature deviations from the ISA standard atmosphere and wind effects are allowed for in the current algorithm, but assumed to be altitude and position independent. Weather is not considered as a variable.

## Structure of the Algorithm

Two distinct but interacting programs are used in determining an optimum flight profile; namely, the mathematical MVO program and flight profile generator.

The MVO routine, developed at the Royal Aircraft Establishment,[2] is designed to minimize a function $F(x)$ of $n$ variables subject to

| | |
|---|---|
| Equality constraints | $C_j(x)=0$ |
| Inequality constraints | $C_j(x)>0$ |
| Variable bounds | $xL_i<x_i<xU_i$ |

$F$ and $C$ are differentiable functions of $x_i$ and derivatives $\partial F/\partial x_i$ and $\partial C/\partial x_i$ are approximated by central differences.

Iteration proceeds by a sequence of quasi-Newton steps, such that

$$x^{(k+1)}=x^{(k)}+\alpha^{(k)}p^{(k)} \qquad p^{(k)}=-H^{(k)}g^{(k)}$$

where $g^{(k)}$ is the vector of first partial derivatives, and $H^{(k)}$ a positive definite estimate of the inverse of the second derivative matrix at $x^{(k)}$. $\alpha^{(k)}$ is determined by searching for a local minimum along direction $p^{(k)}$. Constraints are accounted for by an "ideal" penalty function approach.[3]

This MVO program has been used successfully by aircraft manufacturers in the past for the optimization of aircraft designs.[4] Its applicability to flight profile optimization is a new research area.

The flight profile generator routine is used to analyze a three-phase flight profile as shown in Fig. 1, consisting of climb, cruise, and descent. Each phase is split into a number of segments, typically 10, to provide the necessary detail in terms of variations in mass, air density, etc. An average performance for each segment is then calculated, using the control variables for the relevant phase.

### Variables

During the climb, the IAS ($V_A$), throttle setting ($T_A$), propeller rpm ($N_A$), and distance traveled ($S_A$) are the controlling variables.

The assumption of a constant IAS implies that the true airspeed is increasing continuously. Hence, the aircraft is undergoing an acceleration, and some energy is channeled thereto rather than to a height gain. A correction to the climb rate[5] is used in each climb segment to account for this effect.

During the cruise a constant altitude will be maintained, thereby reducing the controlling variables by one. It is possible to specify either the IAS or throttle setting, and then the other variable becomes a function of time, altering continuously as aircraft mass is reduced. There is little difference between the two methods—one implying that the pilot will be making slight power adjustments and the other implying that the aircraft will be retrimmed as fuel is burned. The method of constant IAS is used in the present program for reasons of computational speed and simplicity. Cruise IAS ($V_B$) and propeller rpm ($N_B$) are, therefore, chosen as the control variables for this phase.

Descent may be treated in the same way as the climb, using throttle ($T_C$), IAS ($V_C$), rpm ($N_C$), and distance ($S_C$) as the control variables. Note that, for a given stage distance, $S_B=S_{\text{stage}}-S_A-S_C$.

It is frequently necessary to conduct the descent at a constant rate due to cabin repressurization constraints or lack of pressurization capability. The rate of descent (ROD) is then used in place of the throttle as a control variable with a maximum allowable value of, say, 2.54 m/s (500 fpm).

The fuel load burned during the entire flight is also treated as an optimizable variable, as well as being the objective function in most cases. This ensures that only the minimum necessary fuel is loaded on board (diversion and holding fuel is added as a separate quantity), whereas the classical approach requires that a fixed-mass airplane be specified at the starting point before the optimization can commence.

As each flight phase has an associated distinct value of IAS, there occurs a discontinuity in energy when switching from climb to cruise and from cruise to descent. The correction applied to cater for this effect consists of placing the aircraft at a slightly different height when switching from one phase to another. Figure 2 shows the change from cruise to climb, and the adjustment in height, which is given by

$$\Delta h = \tfrac{1}{2}\,(V_B^2 - V_A^2)/g$$

where the speeds are now TAS. The actual flight path followed by the pilot (shown by dotted line) would be accelerated and dependent on piloting technique.

### Constraints

When a given vector of control variables is used in conjunction with the flight profile generator, the aircraft will climb, cruise, and descend according to this vector. However, there is no control over the vertical position of the final point on the flight profile, when the specified horizontal stage distance is reached. Therefore, an equality constraint (EC) is introduced to ensure that the final height reached is equal to the required value, e.g., the elevation of the destination airport plus 305 m (1000 ft).

An inequality constraint (IC) specifies that the cruise distance must not be less than a given value, e.g., 10 km. This ensures that the cruise segments do not become too small, thereby causing problems in the iterative computation of fuel burn. An additional benefit is that passenger comfort considerations would weigh against a strict saw-tooth profile (i.e., a climb-and-descent-only flight).

Inequality constraints are also used to ensure that the power setting during the cruise (and during the descent at constant ROD) are within the engine operating limitations. The throttle during the climb (and during descent at constant $T_C$) can be automatically restricted to lie within a given range since it is a

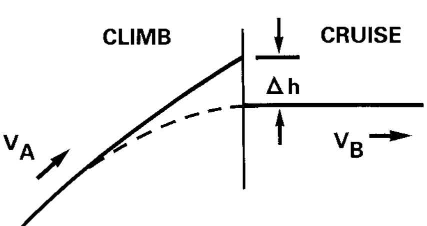


Fig. 2 Energy correction.

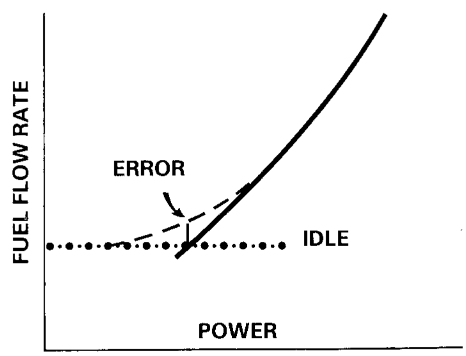


Fig. 3 Blending of two functions.

control variable. Similarly the indicated airspeeds at all three phases are optimizable and restricted to values between $1.2V_{stall}$ and $V_{no}$ (maximum normal operating IAS).

An optional inequality constraint can be used to limit the cruise altitude to below a certain value, representing pressurization or ATC limitations.

Since the fuel load is both an optimizable variable and the objective function, it is necessary to introduce a constraint dictating that the final aircraft weight must be equal to the sum of the operating empty weight (OEW), payload, and reserve fuel load. This effectively prevents the program from searching for an ever-decreasing fuel mass.

Two further optional equality constraints are available. The first can dictate that the total flight time be equal to a given value and is useful when tight scheduling requirements have to be met or when wishing to study tradeoffs between fuel usage and time-dependent operating costs. The second constraint can be used to specify that an aircraft must arrive at a route position at a given time. This can be useful in analyzing ATC requirements, e.g., compulsory reporting points which have to be used by many aircraft at different times.

### Aircraft/Powerplant Modeling

The fundamental aerodynamic characteristics of the aircraft must be incorporated in the program in the form of a lift-drag polar. A parabolic polar representative of a current short-haul aircraft is used. The power available is required as a function of altitude, airspeed, and propeller rpm to determine throttle settings. In the case of the relevant turboprop, linear and parabolic fits provide adequate results. Fuel consumption data are included in the form of surface-fitted Chebyshev polynomials[6] giving the fuel flow rate as a function of power and altitude.

Occasionally it becomes necessary to represent a quantity through more than one equation. For example, the fuel flow rate will be the greatest of the surface-fitted value and an "idling" value which gives adequate combustion stability. Such a situation cannot be represented through conditional "if" statements, because this would create discontinuities in the derivatives $\partial(\text{fuel flow})\partial x_n$ which would upset the optimization routine. A blending routine is used to provide a smooth curve joining the two relationships in the region of discontinuity.[7] The error introduced by this technique can be made as small as desired (Fig. 3). "Flat-rated" torque limits are dealt with in a similar fashion.

Propeller characteristics are modeled using two surface-fit polynomials which co-relate the nondimensional thrust coefficient ($C_T$), power coefficient ($C_p$), and advance ratio ($J$) as follows:

$$C_T = f_1(C_p, J) \qquad C_p = f_2(C_T, J)$$

where

$$C_p = \text{Power}/(N^3 D^5_{\text{prop}} \rho)$$
$$C_T = \text{Thrust}/(N^2 D^4_{\text{prop}} \rho)$$
$$J = VN/D_{\text{prop}}$$

The form $f_1$ is used during the climb (and descent at constant throttle) where the power setting is specified and the thrust must be calculated. The other equation ($f_2$) is used during the cruise (and descent at constant ROD) when power depends on the values of the other controlling variables and must be calculated from the thrust (drag) through a reverse fit. A simple expression is used to calculate the residual jet thrust separately.

The program is capable of optimizing flights sequentially to represent multistage operations. The last stage is analyzed first and the fuel burned is then allocated as a weight penalty to the previous stage, and so on. This technique can also be used to model the last stage as a diversion stage for the calculation of contingency fuel. It is also possible to account for the effects of winds and variable ISA conditions.

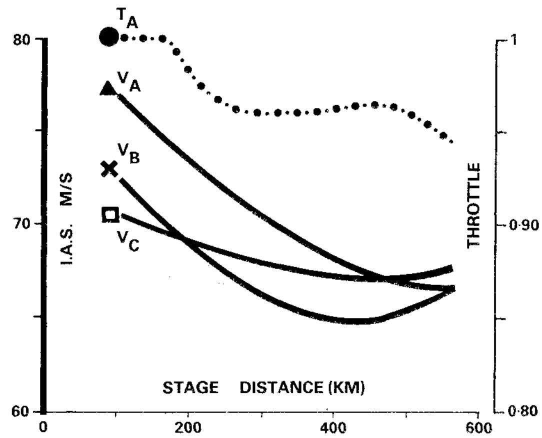


**Fig. 4a Variation of control parameters (7620-m altitude constraint and 2.54-m/s ROD).**

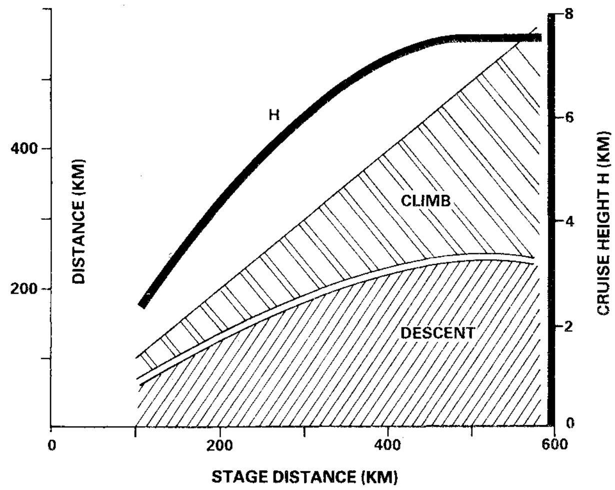


**Fig. 4b Variation of cruise height and relative proportions of flight phases (7620-m altitude constraint, 2.54-m/s ROD).**

## Results and Parametric Studies

A sequence of parametric studies was conducted using fuel burn as the objective function to be minimized. The stage length flown was varied between 100 and 600 km (typical values for short-haul operations) and the optimum values of the control variables corresponding to the various stage lengths were examined.

Figure 4 shows a minimum-fuel flight profile study incorporating a 7620-m (25,000-ft) cruise altitude constraint with the ROD control variable limited to a value of 2.54 m/s (500 fpm). It is seen in Fig. 4a that, in general, the optimal climb IAS ($V_A$) is higher than the optimal descent IAS ($V_C$). This result has been previously confirmed by Stengel and Marcus[8] and Erzberger et al.[9] for jets. It is standard practice for operating manuals of existing commuter aircraft to recommend descent speeds higher than climb speeds for minimum fuel burn, although this may be mathematically incorrect.

The optimum profiles are of the "saw-tooth" type, i.e., a climb-descent with the artificial 10-km minimum cruise constraint active (Fig. 4b). Therefore, the maximum altitude attained increases with stage distance until the 7620-m constraint is encountered at a distance of approximately 470 km.

For propeller-driven aircraft, the value of IAS that maximizes the climb rate reduces with altitude. This may explain why the optimum value of $V_A$ also reduces with stage distance, as the maximum altitude during the flight increases.

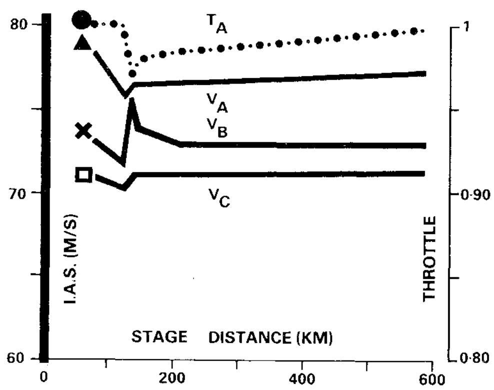


**Fig. 5a Variation of control parameters (3050-m altitude constraint, 2.54-m/s ROD).**

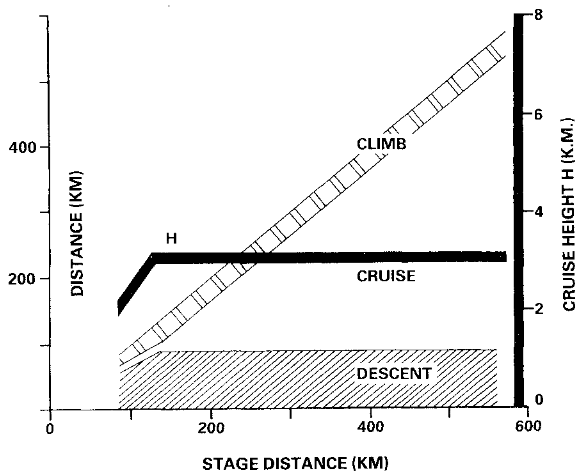


**Fig. 5b Variation of cruise height and relative proportions of flight phases (3050-m altitude constraint, 2.54-m/s ROD).**

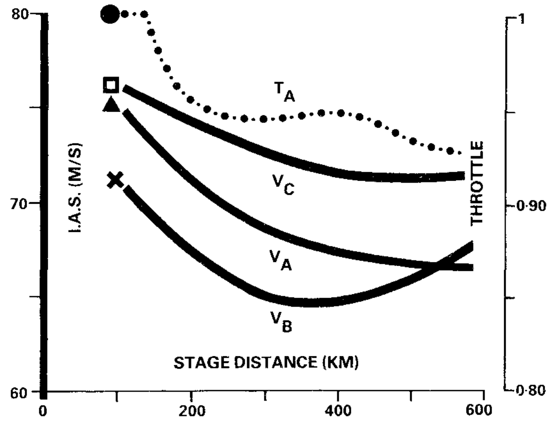


**Fig. 6a Variation of control parameters (7620-m altitude constraint, free ROD).**

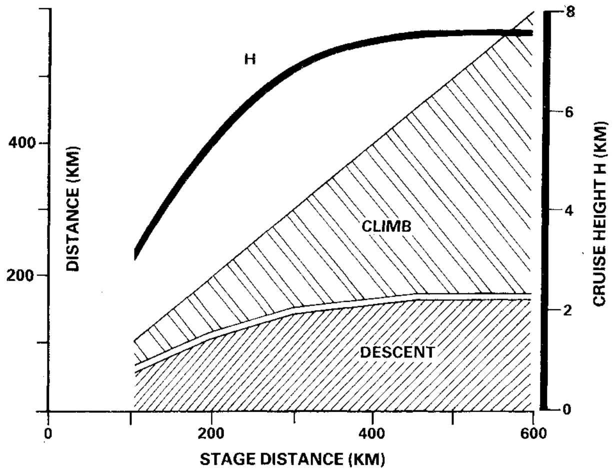


**Fig. 6b Variation of cruise height and relative proportions of flight phases (7620-m altitude constraint, free ROD).**

In contrast, the descent IAS is significantly less dependent on stage distance since optimum descents at low power settings would favor operation at near maximum $L/D$ (constant IAS). The climb throttle is fully open initially, but after 200 km it drops to about 96%, with a change in slope when the 7620-m limit is reached.

For small stage lengths, a greater proportion of the total distance is covered during the descent than during the climb. With increasing stage length, the proportion of the climb increases. Erzberger et al.[9] produced similar results for jet aircraft.

Figure 5 shows the results of a study similar to the above, but with the maximum altitude constraint reduced to 3050 m (10,000 ft). Initially, the results are the same until the altitude constraint is reached at a distance of approximately 130 km. The values of the control variables for the climb and descent portions then remain substantially independent of stage distance, and the cruise segment becomes significant. The discontinuities in the control variables apparent in Fig. 5a are the consequence of a "multiple event" occurring when the 3050-m constraint becomes active. This forces the 10-km cruise distance constraint to become inactive and pushes the climb throttle off its maximum value. Such behavior is characteristic of MVO routines and is described in detail by Edwards.[10] Figure 5b shows the relative proportions of distance flown in each flight phase.

Figure 6 shows a study again using the 7620-m constraint but allowing the ROD to find its optimal value for any given stage length. The results are similar overall to the 7620-m/2.54-m/s limit study with the exception that descent speed now becomes higher than climb speed. The optimal values of the ROD are in the region of 4.0 m/s. Also because of the steeper descent the proportion of climb to descent increases (compare Fig. 6b with Fig. 4b).

It is possible to use the descent throttle ($T_C$) instead of the ROD as a control variable, as shown in Fig. 7. Descent throttle tends to assume idling values, and descent speed remains substantially independent of stage distance. The most interesting feature of this study is that optimum cruise altitudes are found, i.e., the profiles are not of the "saw-tooth" type. The cruise altitude increases with stage distance as the proportion of the cruise increases to total distance.

Fuel usage against stage distance is plotted in Fig. 8 for all of the above studies. Minimum fuel burn is achieved when the highest altitude limit (7620 m) is used and the optimum value of ROD to 2.54 m/s. Using the throttle ($T_C$) to control the descent results in a further fuel penalty. Therefore, it appears that ROD-controlled profiles will be closer to the mathematical optimum than throttle-controlled profiles. When throttle-controlled descent is selected, the program tends to choose idling values and, therefore, the aircraft descends under near maximum $L/D$ conditions (to minimize energy loss). Consequently, the flight-path angle is roughly constant, i.e., a straight-line descent. When ROD is used the flight-path angle steepens as the aircraft descends, i.e., the flight path is concave downward which approximates the results of Erzberger et al.[9] more closely for mathematically optimum profiles.

Using the 3050-m altitude constraints with a 2.54 m/s ROD results in a substantially greater fuel burn. This is the operationally representative case for unpressurized aircraft.

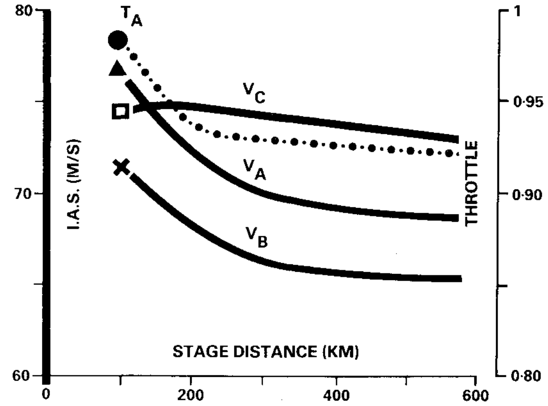


Fig. 7a Variation of control parameters (7620-m altitude constraint, free $T_C$).

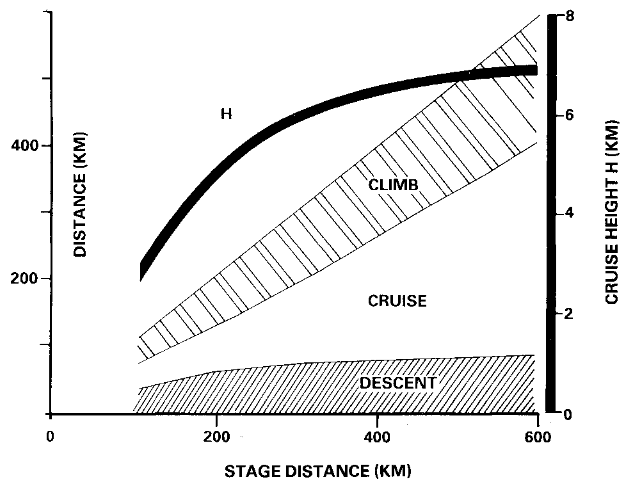


Fig. 7b Variation of cruise height and relative proportions of flight phases (7620-m altitude constraint, free $T_C$).

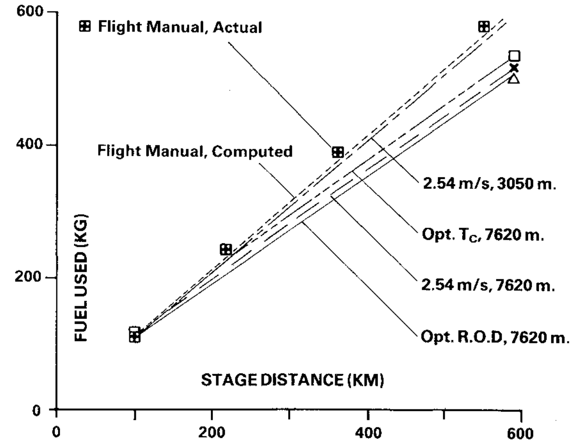


Fig. 8 Variation of fuel burn with stage distance.

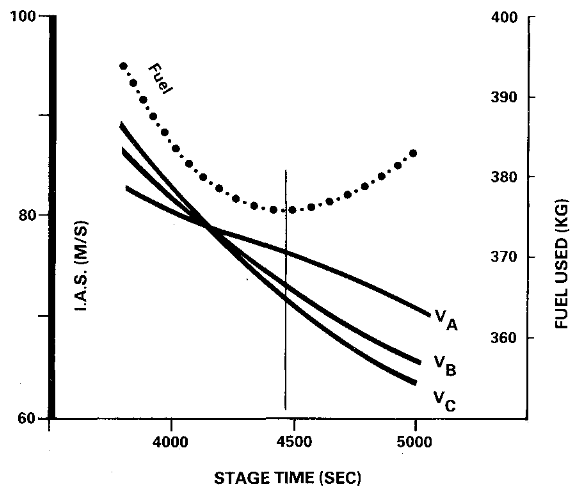


Fig. 9 Effects of fixed time of arrival.

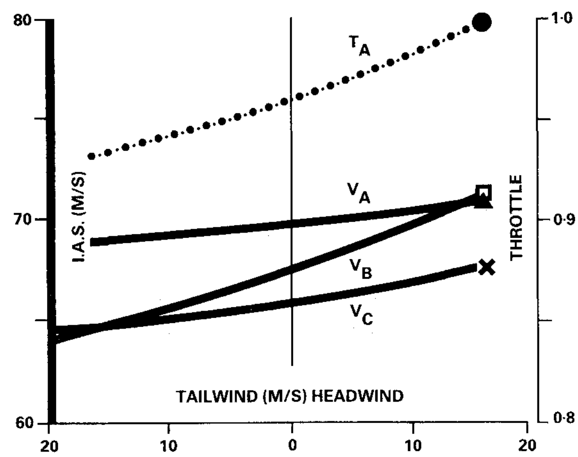


Fig. 10 Wind effects.

In order to check the accuracy of the methods, the program was made to fly several profiles such as described in the aircraft operations manual. The computed fuel burn was between 1.5 and 4% less than given in the manual, but this discrepancy is believed to be attributable to conservative rounded-up figures used in the manual to account for operationally degraded engines.

A fuel saving of approximately 1.5% can be achieved relative to flight-manual economy techniques (computed fuel burns) by flying the optimum profiles which incorporate the 3050-m/2.54-m/s operational constraints. Fuel savings that would be possible for a pressurized airplane are on the order of 15% or more.

A minimum fuel study was conducted using the fixed-time-of-arrival constraint. An arbitrary stage distance was chosen (370 km) and after determining the flight time (4480 s), a sequence of minimum fuel runs was made with fixed arrival times. Figure 9 shows the effective tradeoffs between fuel burn and flight time. It is noted that, although climb speed is greater than descent speed for the unconstrained case, the situation is reversed as the flight time is reduced.

An example of the effects of wind is given in Fig. 10. The optimal values of the throttle setting and IAS increase when a headwind is present, and descent speed is seen to alter more sharply than climb speed.

Wind has a very significant effect on fuel burn. A typical value of 15 m/s of headwind, which would be common during normal operations, increases fuel consumption by 18% for the study shown in Fig. 10. A tailwind of similar magnitude reduces the fuel consumption by 14%.

Some studies were conducted to examine the effects of temperature, varying this from ISA + 20 to ISA − 20°C. It was found that the fuel consumption was practically unaffected by deviations from the ISA standard. Although the fuel flow rate is slightly higher in the "hot" cases, the trip time is reduced because of increased values of true airspeed, and the overall effect on the total fuel burn is negligible. Flight manual data agree with this observation. Because of the combination of decreased trip time and unchanged fuel burn, the surprising conclusion can be reached that hot-day operations would be cheaper in terms of operating costs.

In all of the preceding studies, propeller speeds are operationally restricted between 1400 and 1200 rpm, except for takeoff. The program selects the maximum value during the climb and the minimum during the descent, and optimum cruise rpm's on the order of 1300 are found.

## Conclusions

Multivariable optimization techniques have been used to evaluate short-haul flight profiles. The methods are most suitable for applications to propeller-driven aircraft, and result in operationally realistic flight profiles which can be flown manually. The performance of the algorithm was verified against flight-manual data, and a 1.5% reduction in fuel burn was found to be possible. A variational/dynamic analysis program capable of handling propeller-driven aircraft was not available for comparison of the calculated suboptimal trajectories with the true optimum.

Any mathematical objective function can be used, although only minimum fuel studies have been discussed in this paper. These studies show that the climb and descent segments are significant in relation to the total fuel burn and that substantial penalties can result if altitude constraints are imposed due to operational reasons. Equality and inequality constraints can be incorporated in the algorithm without altering its basic structure.

## Acknowledgments

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## References

[1]Shultz, R. L. and Zagalsky, N. R., "Aircraft Performance Optimization," *Journal of Aircraft*, Vol. 9, Feb. 1972, pp. 108-114.

[2]Purcell, A. G., "An Alternative FORTRAN Program for Constrained Optimisation," Royal Aircraft Establishment, Bedford, England, U.K., TM Math-Comp 7908, March 1980.

[3]Purcell, A. G., "FORTRAN Programs for Constrained Optimisation," Royal Aircraft Establishment, Bedford, England, U.K., TR 77142, Sept. 1977.

[4]Chacksfield, J. E., "Optimisation—A Useful Tool for Aircraft Design Studies," *Bulletin of the Institute of Mathematics and Its Applications*, Vol. 18, Sept./Oct. 1982, pp. 170-177.

[5]"Performance" Data Sheets Engineering Sciences Data Unit, London, Vols. 1-3.

[6]Numerical Algorithms Group (NAG) Subroutine E02 CAF., Nottingham, England, U.K., Dec. 1978.

[7]Edwards, B., "BLEND—A Function Equal to the Greater of Two Other Functions with a Fairing Where They Intersect," Unpublished note, Royal Aircraft Establishment, Bedford, England, U.K., July 1977.

[8]Stengel, R. F. and Marcus, F. J., "Energy Management for Fuel Conservation in Transport Aircraft," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-12, July 1976, pp. 464-470.

[9]Erzberger, H., McLean, J. D., and Barman, J. F., "Fixed Range Optimum Trajectories for Short-Haul Aircraft," NASA TN D8115, Dec. 1975.

[10]Edwards, B., "The Use of Computer Based Optimization Methods in Aircraft Studies," AGARD CP 280, 1979.